UK Maths Trust

Intermediate Mathematical Olympiad

Maclaurin paper

Thursday 20 March 2025

England & Wales: Year 11 | Scotland: S4 | Northern Ireland: Year 12

These problems are meant to be challenging.

Try to finish whole questions even if you cannot do many; you will have done well if you hand in a complete solution to two or more questions.

Instructions

1. Time allowed: **2 hours**.
2. **Full written solutions – not just answers – are required**, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
3. **One complete solution will gain more credit than several unfinished attempts**.
4. **Each question carries 10 marks**.
5. The use of rulers, set squares and compasses is allowed, but **calculators and protractors are forbidden**.
6. Start each question on an official answer sheet on which there is a **QR code**.
7. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code. **Please do not write your name or initials on additional sheets**.
8. **Write on one side of the paper only**. Make sure your writing and diagrams are clear and not too faint. (Your work will be scanned for marking.)
9. **Arrange your answer sheets in question order before they are collected**. If you are not submitting work for a particular problem, please remove the associated answer sheet.
10. To accommodate candidates in other time zones, please do not discuss the paper online until **12pm GMT on Saturday 22 March**, when the video solutions will be released.
11. Do not turn over until told to do so.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

challenges@ukmt.org.uk *www.ukmt.org.uk*

Advice to candidates

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
- ◇ *Your answers should be fully simplified, and exact. They may contain symbols such as* π*, fractions, or square roots, if appropriate, but* not *decimal approximations.*
- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◇ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do* not *hand in rough work.*

1. A cuboid box has height 10 and a square base of side length x. It is put in a cylindrical water tank with base a circle of area 360. The cuboid stays at the bottom when water is added.

When the box is in the tank of water with a square face against the bottom of the tank, the water reaches the top of the box.

When the box is in the tank of water with a non-square face against the bottom of the tank, the water level drops by 1.

The box is removed from the tank. How much further does the water level drop then?

2. Alice and Charlie play a game, taking it in turns with Alice going first.

On a blackboard is written a three-digit number. If the current number on the blackboard is n, a move consists of choosing a non-zero digit, k, of n and replacing n with $n - k$ on the blackboard. This is repeated until the number 100 is written on the blackboard.

The player who writes the number 100 wins.

(a) If the starting number is 125, Alice can always win. State Alice's first move and how Alice responds to whatever move Charlie makes at each stage.

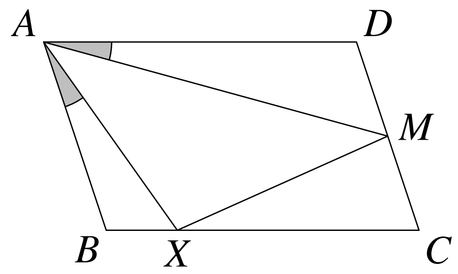
(b) Find, with proof, for which starting values Charlie has a winning strategy.

3. The digits of 2024 have the property that $7^2 + 7^0 + 7^2 + 7^4 = 2500$ ends in two zeroes.

Find how many sets of solutions there are for single digits a, b, c and d so that $7^a + 7^b + 7^c + 7^d$ ends in two zeroes.

Note that $(a, b, c, d) = (2, 0, 2, 4)$ and $(a, b, c, d) = (0, 2, 2, 4)$ would be two different sets of solutions.

4. In a parallelogram $ABCD$, M is the midpoint of DC. There is a point X on BC such that $\angle MAX = 30°$, $\angle BAX = \angle MAD$ and $AX = XM$. Find the ratio of $BX : XC$.



5. Let m and n be positive integers such that m divides $n + 20$ and n divides $m + 25$. What is the maximum possible value of $m + n$?

6. There are fourteen rectangular tiles with sides of length 3 and 4 and one square tile with sides of length 1 to tile a square floor which is 13 by 13.

(a) Show that this is possible if the 3 by 4 tiles are cut into two 3-4-5 triangles.

(b) Prove that this is impossible without cutting any tiles.